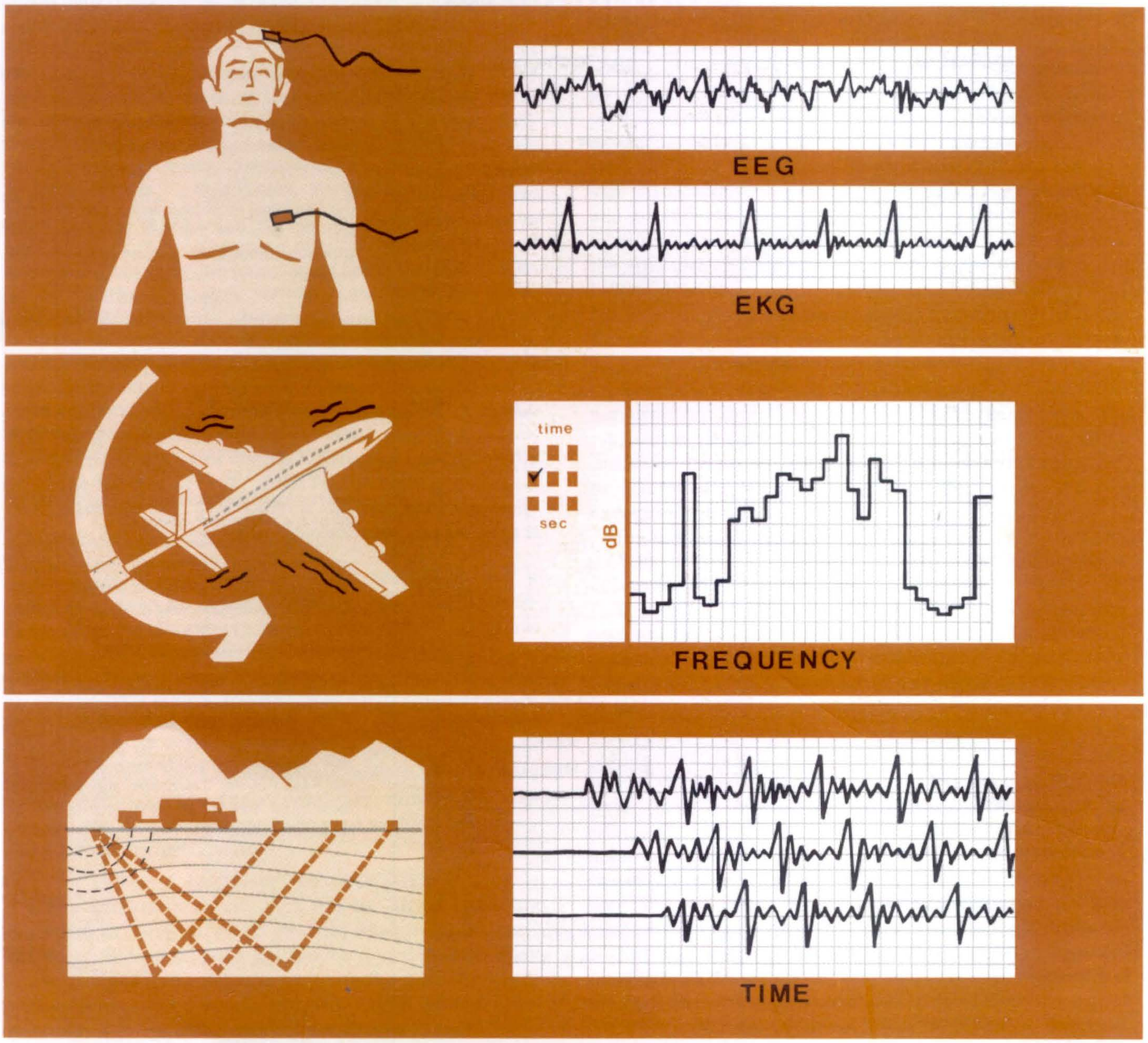
THE
GENERAL RADIO

Experimenter

VOLUME 44
NUMBERS 7, 8, 9
JULY – SEPTEMBER 1970



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The *General Radio Experimenter* is mailed without charge to engineers, scientists, technicians, educators, and others interested in the instruments and techniques of electrical and electronics measurements. Address all correspondence to Editor, *General Radio Experimenter*, General Radio Co., Concord, Mass. 01742.

The Cover illustrates several of many successful applications of signal analysis in real time. The ability to match action with reaction, while events are taking place, is an excellent cost-reduction measure. It also permits knowledgeable experimental control during actions, as contrasted with hindsight decisions forced upon us by delayed data. The end result is an intelligent, economical approach to research, development, test, and production problems.

Complexity in the mathematical treatment of circuits and systems has been reduced in the past ten years. Small computers make possible our present-day electronic analyzers, programmed to apply all sorts of mathematical analytical techniques to design and test projects that will save engineering time and dollars. As part of simulation devices, the analyzers also can perform real-time functions in imitation of control by humans, thereby helping to reduce hazards to human operators.

The majority of design and test engineers are relatively too unskilled in the assembly and utilization of available instrumentation to create something which resembles, even remotely, the complex analytical test sets presently sold. This lack of skill is not in the mechanics of assembly but in the *understanding of the techniques involved.*

There are, undoubtedly, numerous readers who bravely entered Norbert Wiener's classroom at M.I.T. to study the secrets of effective communication and of communication analysis. If, like this Editor, you departed from the seminar with a modicum of new knowledge and a plentitude of frustration, you will welcome another chance to tackle a phase of signal analysis, helped by the less difficult presentation on the next page. After that, perhaps, you will be ready for the age of the Fast Fourier Transform and associated techniques and for the instruments that incorporate them.

C. E. White

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Editor

Signal Analysis with Digital Time-Series Analyzers

Although time-series analysis techniques are not new, they have not been used extensively because of the lack of suitable equipment, and therefore are not familiar to many engineers. The literature describing these techniques generally has been written for an academic audience and has been often clouded in abstract mathematics rather than being presented in user-oriented terms. This is an unnecessary obstacle to place before the potential user, because there is no more need to be a mathematician to use a time-series analyzer than there is to use a spectrum analyzer. In this article, the basic principles will be presented in non-rigorous physical terms as much as possible and will be illustrated with examples from several typical applications.

Analyzing electrical signals is a fundamental problem for engineers and scientists in all fields, whether they are working in the research and development laboratory, on the production line, or in the field. Whatever the physical, biological, or chemical system being studied, the basic phenomena can usually be converted into electrical signals by suitable transducers and analyzed to provide fundamental information about the system producing the signals.

Until recently, only a few basic analysis procedures were used, because of the limitations of the technology available for building test equipment. The basic signal-analysis procedure, other than direct observation of the signals on an oscilloscope, has been power-spectrum analysis, and will probably continue to be so. However, there are many other basic analysis procedures that would provide much more valuable information about the system under test, but they have not been available with the performance required for most practical problems. Even spectrum-analysis equipment, although serving adequately for a wide variety of problems, has severe limitations due to the analog-circuit technology available.

In the last decade, the availability of general-purpose computers allowed the advanced signal-analysis techniques that had been pioneered by Wiener, Lee, et al, to be developed to practical computational routines and were applied successfully to many data-analysis problems. They do not, however, help the experimenter or analyst who requires results rapidly so that he can interact with his system. The cost of computer analysis for this type of problem is also very high, especially when the loss of engineering time while awaiting results is taken into account.

The revolution in digital-processing technology that is now taking place has brought advanced signal-analysis techniques into the laboratory. This advance enables economic production of instrumentation systems which contain all of the computational power required for fast, accurate, signal processing. The TD 1923 Time-Series Analyzers, soon to be introduced by Time Data and GR, are the most advanced and complete line of this type of signal-processing equipment available. Besides their computational ability, they contain signal conditioning and display capability, with flexible, simple controls that allow them to be used as easily as conventional laboratory instruments.

WHERE AND WHY THEY'RE USED

Every process in nature gives rise to "signals" that are amenable to analysis by time-series analysis techniques. Therefore, the list of potential applications is endless. They can be grouped into categories based on the type of processing that is used and on the information that is desired. Some examples from several categories are illustrated on the front cover and described below.

Consider first some examples in the field of structural mechanics, where the basic quantities to be analyzed are mechanical vibrations in aircraft, automobiles, buildings, etc. These vibrations may be caused by many external forces such as wind, engine combustion and rotation, road roughness, earth tremors, and impact. The designer would like to determine such things as the source and transmission paths of the vibrations and the expected stresses and displacements at various points, by analyzing the signals from points on the structure. For instance, during the testing of an automobile, an objectional vibration may be found to exist in the passenger compartment. Time-series analysis techniques will determine if the vibration is coming from the engine, the road, or from the wind. In addition, the structural members that provide the transmission path for the vibration can be identified and suitably modified to filter out or to isolate the unwanted vibrations. The analysis system can even be used to simulate the structure and the external forces to test the new design before it is committed to production. Vibrations coming from the engine can also be analyzed to determine its condition. Failures may be predicted, or at least interpreted, from such a "signature analysis."

Another example is the determination of the flutter characteristics of aircraft in flight or in wind-tunnel tests. The resonant frequencies and damping of the various vibration modes of the aircraft are measured as a function of airspeed, to determine if there are any conditions that will produce excessive or unstable vibrations.

In the third example, rumbling vibrations, caused by automobiles driving across a bridge, are analyzed to pinpoint areas of high or unexpected loadings.

The field of biomedicine has already provided many important applications but it has only now begun to make use of sophisticated analysis techniques. The nervous system of the human body naturally produces electric signals whose characteristics are indicative of the condition of various parts of the body. The most common example is the electrocardiograph (EKG) signal, which provides information about the condition of the heart. Electroencephalograms (EEG's), or brainwave tracings, are analyzed to study brain damage and the effects of various stimuli and drugs on the brain. The techniques now being used by clinical physicians to analyze these signals are very primitive compared to those which can be used. The widespread use of time-series analysis techniques in medicine should increase as they become better understood.

Another application is geophysical exploration, an echo-ranging application that is in the same category as radar and sonar. A vibration signal is transmitted and echos are received from the reflecting geological strata. Comparison of the received and transmitted signals determines the time delay between them and, therefore, determines the location of the reflecting surfaces. Poor signals can be enhanced through analysis techniques, and a profile of the underground terrain obtained.

TIME-SERIES ANALYSIS

The preceding problems can all be solved with various combinations of the basic computational routines that constitute the tools of time-series analysis. By a time series or signal, we simply mean a succession of data values resulting from or simulating a physical process. The data can be amount of rainfall, earthquake tremors, electric voltages from a brain, or any other physical process. Time is usually the independent variable, but not always. The common thing about all these records is that the successive data values in each of the time series are related in some way to the other values in that time series, and perhaps to values in some other time series. These relationships may be deterministic or statistical. Time-series studies analyze these relationships so that the physical process can be better understood. This process can then be modeled, or simulated, either mathematically or physically.

The basic signal-analysis procedures required in a practical instrument can be grouped into three measurement categories, as shown in the table below. The processing functions must generally be fast enough so that data are processed in real time, that is, as fast as they are being acquired. Real-time processing also is necessary because the number of input data samples is usually much larger than the memory capacity of any practical machine. In a typical measurement, the input signal may be sampled for 10 minutes at a rate of 100,000 samples/second. The total number of input samples becomes astronomical but, after being processed by the appropriate analysis procedure, the data may have been reduced to only 100 numbers.

Basic Signal-Analysis Procedures

A. Measurement of Similarity
   1. Correlation analysis
   2. Spectral analysis
   3. Filtering

B. Measurement of Waveforms
   1. Ensemble averaging

C. Measurement of Statistical Distributions
   1. Probability density functions (amplitude distribution)

The product of the processing operation is often the final result desired. Frequently, however, some further processing must be performed on this result to put it into the form most suitable for showing the information being sought. Other processing functions allow this by performing basic arithmetic operations, coordinate transformations, smoothing operations, and time/frequency transformations.

MEASUREMENT OF SIMILARITY

The most important time-series analysis tools are those that give a measure of the similarity between signals. Spectral analysis, correlation analysis, and filtering provide this information. In fact, they provide the same measure of similarity, based upon the mean-squared difference between the signals, but present the results in different forms. Although spectral analysis and filtering are more familiar to most people than correlation, the latter offers more insight into the concept of measurement of similarity and therefore will be discussed first.

C. L. Heizman is a graduate of City College of New York (BEE - 1955) and Columbia University (MS - 1956). He was involved in research and product development of integrated circuits, high-speed computers, and data-analysis instruments prior to joining Time/Data Corporation (a GR Subsidiary) in 1968. As Applications Manager at T/D, Mr. Heizman devotes much time to real-time analysis. He is a member of IEEE.

Correlation Analysis

A natural way to compare the two waveforms of Figure 1 is to subtract one from the other, ordinate by ordinate, square each term to give quantities that are porportional only to the magnitude of the difference, and then to sum all the squared difference terms to obtain a single number that is a measure of the similarity. This number, when normalized by the number of independent measurements, is the mean-square difference. It can then be calculated for various displacements of one signal with respect to the other. By simple algebra, you can show the same information by calculating the correlation, or covariance, function which is the sum of the ordinate-by-ordinate multiplication of the two waveforms.* The result for the two random signals of Figure 1 shows that they are most similar when there is a displacement of 0.7 millisecond between them. If these signals represent the vibration level at two points on a structure, this time is the propagation time for vibrations between the two points. The sign of the displacement indicates the direction of propagation. Measuring time delays in this way leads to many useful applications.

Instead of calculating the cross-correlation function of two different signals as above, you can correlate a signal with itself to give the auto-correlation function. This shows how successive samples of a signal are related.

The auto-correlation functions in Figure 2, for the two random signals of Figure 1, show that successive samples of the upper signal of Figure 1 are more correlated, or are more dependent upon each other, than those of the lower signal.

Another example is shown, in Figure 3, of a sine-wave signal buried in noise. Even though the sine wave is not readily observable in the original signal, the auto-correlation function shows its presence clearly. This ability to detect periodic components in a signal is an important application of the auto-correlation function.

Spectral Analysis

The measurement of similarity is often easier to interpret when the operation is done as a spectral calculation. It is also

*The mean-square difference is

$$\epsilon_D = \frac{1}{N}\sum_n (x_n - y_{n-\tau})^2$$

$$= \frac{1}{N}\left(\sum_n x_n^2 + \sum_n y_n{}^2{}_{-\tau} - 2\sum_n (x_n\, y_{n-\tau})^2\right)$$

All the information about the similarity of the signals is contained in the third term, which is a maximum when the signals are most similar. This term is the correlation or covariance function.

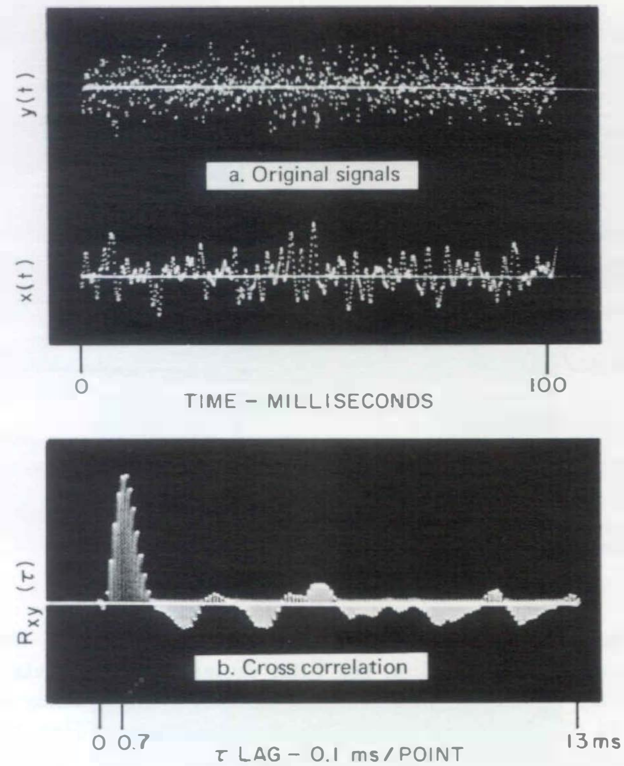


Figure 1. Cross-correlation function of two waveforms.

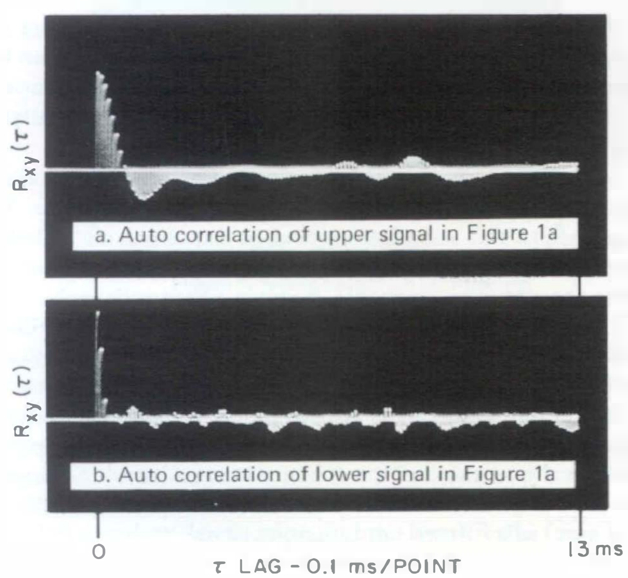


Figure 2. Auto-correlation functions.

more common to see the measurements in this form because in the past spectral analysis was easier to implement with electronic instruments than correlation analysis.

Spectral analysis is based upon the theorem that any repetitive signal can be considered to be the sum of sinusoidal components whose frequencies are integral multiples of the basic repetition frequency. Fourier-transform analysis, familiar to most engineers and scientists, involves the calculation of the amplitudes and phase angles of these components. In principle, the calculation is really a cross correlation of the signal with a sinusoid of each of the possible harmonic frequencies, respectively. The result is always another sinusoid of the same frequency, whose amplitude and phase are proportional to the corresponding component in the signal. In practice, the Fourier-transform calculation is done in a much faster, more direct way by efficient computational methods that have been developed in recent years.

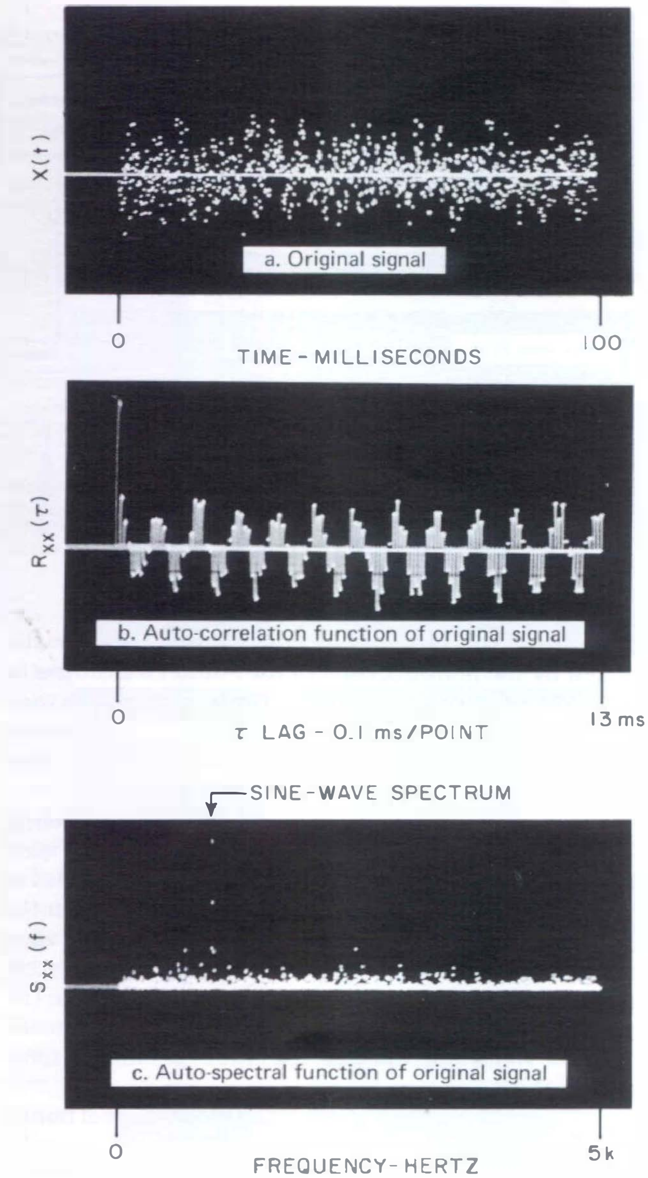


Figure 3. Analysis of sine wave buried in noise signal.

Auto-spectral analysis involves the calculation of the squared magnitude of the Fourier spectrum and is the quantity produced by most spectral analyzers. Because it is proportional to the power of a signal, it is commonly called Power Spectral Density (PSD). It gives exactly the same information as the auto-correlation function; in fact, it is the Fourier transform of the auto-correlation function, and it can be calculated in that way.

A spectral measurement that is not commonly available from analog spectrum analyzers, but which is extremely use-

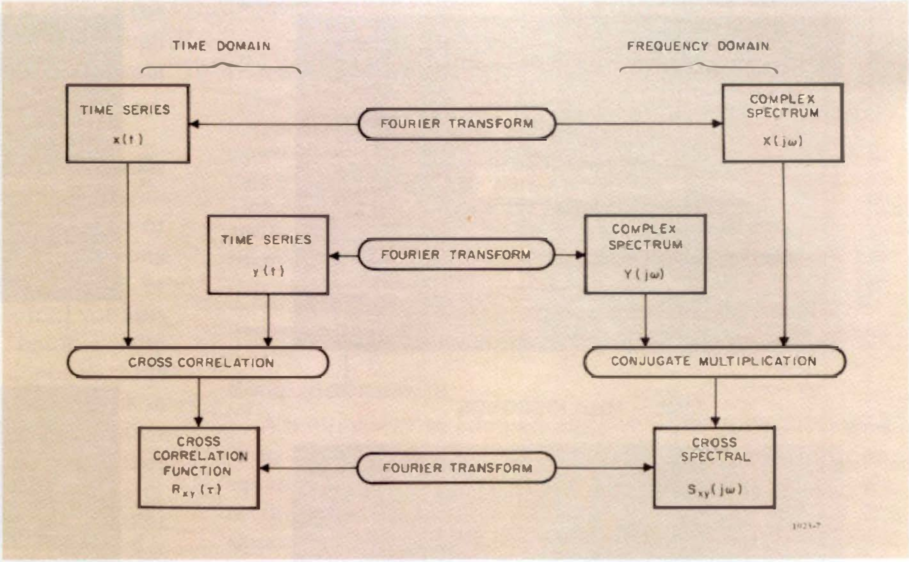


Figure 4. Time/frequency-domain diagram.

ful, is the cross-spectral function. This function is normally calculated by the multiplication of the Fourier transforms of two signals. Like the auto-spectral function, it contains exactly the same information about the similarity of the signals as the cross-correlation function, and it can be calculated from the cross-correlation by the Fourier transform.

The relation between correlation and spectral functions is illustrated by the time- and frequency-domain map of Figure 4. It shows that the correlation function can be calculated in two ways: directly in the time domain, or indirectly in the frequency domain via Fourier transforms. The fast algorithms developed to calculate Fourier transforms have enabled the calculations to be done more quickly via the frequency domain than in the time domain and thus have made the Fourier transform the key operation in modern time-series analyzers.

The auto-spectral function for the sine-wave signal buried in noise is also shown in Figure 3, along with the auto-correlation function. The sine-wave signal is clearly discernible in either function.

Filtering

A filter can be considered as a device that continuously compares an input signal with a stored reference signal and produces a maximum output when the two are most similar. The stored reference signal is the impulse response of the filter, i.e., the response of the filter to a very narrow pulse applied at the input. The criterion for the measurement of similarity is the same as for correlation and spectral analysis: in fact, the calculations are carried out in exactly the same way. Figure 5 shows a filter that was designed to detect the presence of a "chirp" or swept-frequency sine-wave signal buried in noise. This type of signal is used often as the transmitted signal in radar, sonar, and geophysical echo-ranging systems. The output of the filter increases significantly when the signal is present and has the same shape as the auto-correlation function of the signal.

The implementation of filters to "match" a prescribed signal is quite difficult with analog components, and practically impossible if the filter is to be easily variable. With digital implementation, the filter design becomes trivial – merely the specification in either the time or frequency domain of the waveform to be detected – and the filter characteristic can be changed in microseconds.

MEASUREMENT OF WAVEFORMS

Ensemble averaging* is very useful for determining the shape of a signal that is obscured by random noise when the signal is repetitive or when its time of occurrence is known. This latter condition exists when a system is being stimulated in a controlled manner.

An example is the electroencephalograph (EEG) signal produced by the flashing of a light in a person's eye. The responses to this stimulation (evoked responses) are added or averaged together. The signal-to-noise ratio is increased because the signal components, being in phase with each other, will add linearly while the noise components, being random, will partially cancel each other and add at a rate proportional to the square root of the number of averages.

Ensemble averaging is also commonly applied to the measurement of correlation and spectral functions to improve the statistical accuracy of the measurement. Consider the auto-spectral measurement of a short segment (1000 samples in this case) of a filtered random noise signal, as shown in Figure 6. The auto-spectral function itself is also a random function. Averaging the auto-spectral measurements of successive segments of the signal reduces the statistical variations.

MEASUREMENT OF STATISTICAL DISTRIBUTIONS

The amplitude histogram is often the first measurement made in the analysis of random data. You determine it by dividing the amplitude range into many equally spaced levels and by counting the number of times the measured value of the signal is at each level. The histograms of the two random signals of Figure 1 are shown in Figure 7. Besides showing the

*A statistical average evaluated from the probability density of a random process.

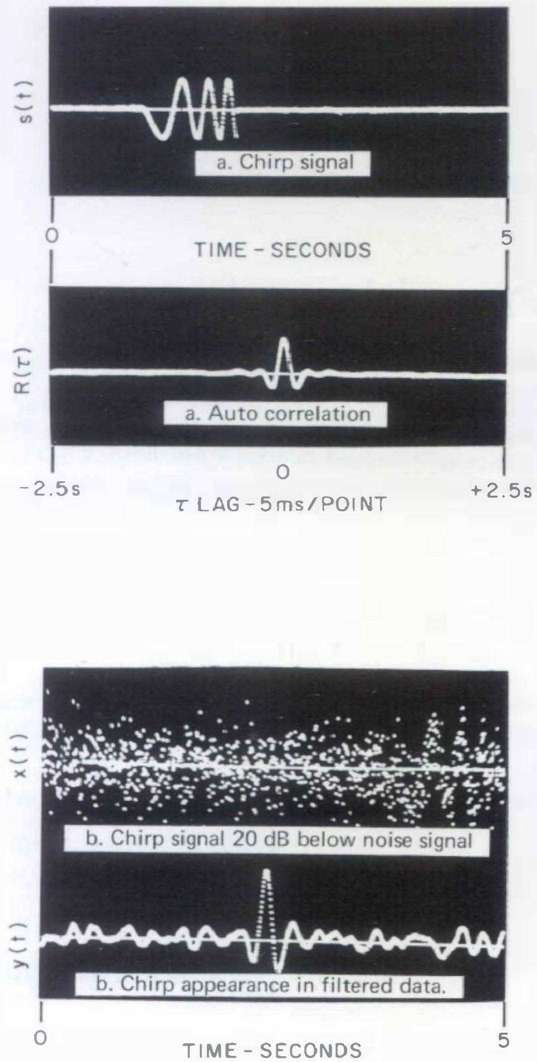


Figure 5. Signal detection by filtering.

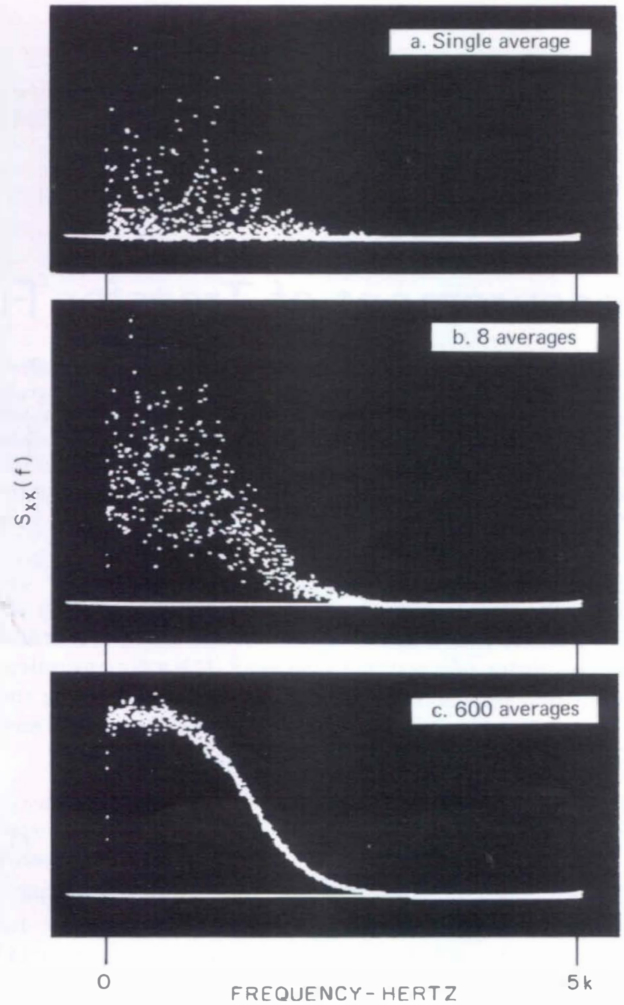


Figure 6. Averaged auto spectrum of band-limited noise.

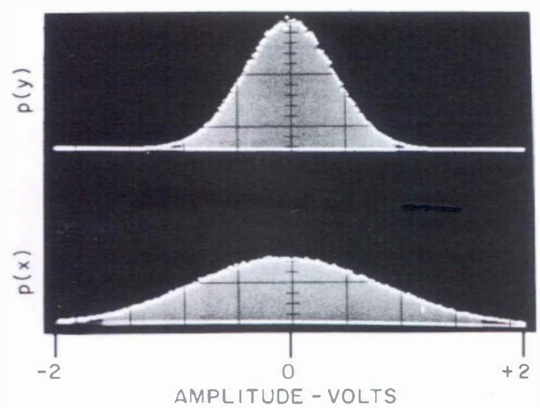


Figure 7. Amplitude histograms of random noise.

extremes of signal amplitudes to be expected, the amplitude distributions give information about the linearity of the relationship between the signals. If these signals represent the input and output of a system under test, the fact that both signals have the same distribution, Gaussian in this case, indicates that the system is probably linear.

From amplitude histograms, the mean value, rms value, and higher-order moments can be calculated.

SUMMARY

The basic tools required for analyzing signals have been discussed, and some of their applications have been given. The Fourier transform is seen to be a fundamental calculation required in digital time-series analyzers for doing correlation and spectral analysis and filtering. Many other calculations often are required, such as coherence function, transfer function*, cepstrum, etc, but these are extensions of the fundamental procedures described and are obtained by further processing. It is now practical, by application of modern digital technology, to build instruments that can accomplish all these functions with the performance required for real-world problems.

C. L. Heizman

*See page 8.

Measurement of Transfer Function and Impedance

The concept of transfer function is widely employed in the analysis and synthesis of linear systems or networks. Conventional instrumentation is well suited to the problem when a simple noiseless single input, single output system is involved, particularly if the rate at which measurements must be made is low. If not, modern FFT (Fast Fourier Transform) techniques provide an efficient means for performing these measurements at lightning speed. This paper derives the best (least mean-squared) transfer-function and impedance estimator, which is shown to involve the ratio of the input-output cross-spectral to the input auto-spectral density.

General

The transfer function of a linear network is a popular and useful descriptor of a system or network. It is a dimensionless quantity that relates the input and output by specifying the gain (or attenuation) and phase shift at all frequencies. Thus, the transfer function of the system shown in Figure 1 is

$$H(f) \angle \theta(f) = \frac{Y(f) \angle \psi(f)}{X(f) \angle \phi(f)} \qquad (1)$$

where

$$H(f) = \frac{Y(f)}{X(f)} \qquad (2)$$

is the gain of the system for a frequency of f Hz as measured by the ratio of the output function $Y(f)$ and the input driving function $X(f)$,

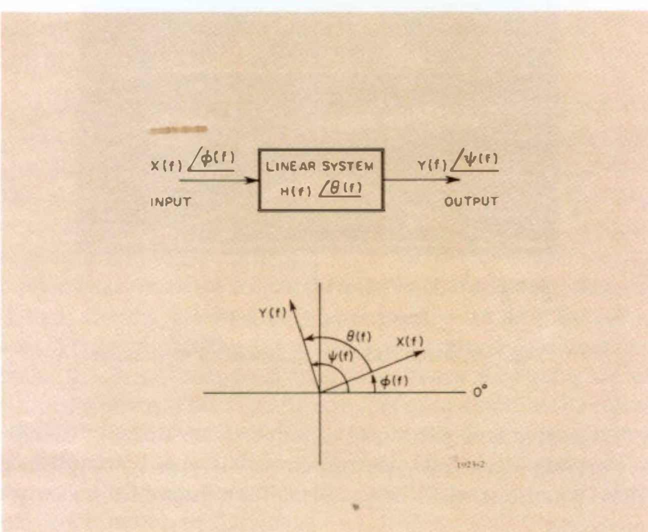


Figure 1. Linear-network derivation.

while

$$\theta(f) = \frac{\angle \psi(f)}{\angle \phi(f)} = \angle \psi(f) - \angle \phi(f) \qquad (3)$$

is the relative phase shift introduced by the system, as measured by the difference in phase angle between the input and the resultant output.

In terms of complex notation, the transfer function is

$$H(f) \exp[j\theta(f)] = \frac{Y(f) \exp[j\psi(f)]}{X(f) \exp[j\theta(f)]} \qquad (4)$$

$$H(f) \exp[j\theta(f)] = \frac{Y(f)}{X(f)} \exp(j[\psi(f) - \theta(f)]) \qquad (5)$$

The transfer function often must be measured for purposes of system analysis or simulation. Simulation is particularly important when the system under study is inaccessible or unwieldy for experimentation. Examples of this type might include simulation of process control systems or subsystems that can only be observed in operation, simulation of the response of space vehicles to transient excitation without running the risk of actual damage, or analyzing and simulating the effective transfer function of a vehicle suspension system. Knowledge of the transfer function would permit a convenient analog to be constructed.

Test equipment capable of providing the necessary measurements has been developed over the years and generally includes a sine-wave generator, voltmeters, and a phase-angle meter as the basic tools. The amplitude of the response and its relative phase angle constitute the necessary measurements. A measurement is usually obtained when the frequency of excitation is varied while the input is kept constant.

Standard measurement techniques of this type are usually adequate and are particularly suitable when the system under observation is

1) Noiseless
2) Has a single input port

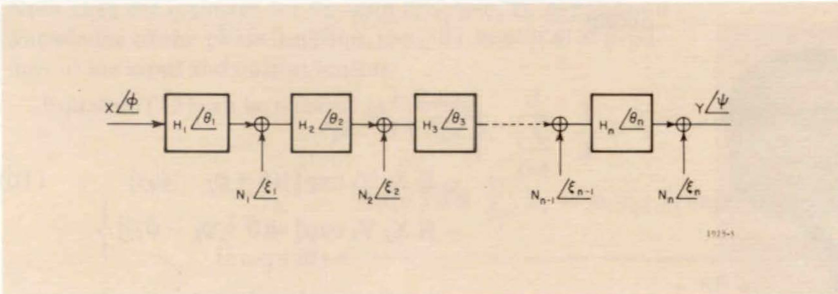


Figure 2. Noisy-network block diagram; multiple inputs.

In other words, if the system is as shown in Figure 2, the input $X\angle\phi$ and the output $Y\angle\psi$ are not simply related by the effective transfer function describing the gain and phase between the X input and Y output, because of the presence of numerous noise sources, propagating toward the output port. We find, therefore:

$$\begin{aligned} Y\angle\psi = {} & X H_1 H_2 \ldots H_n \angle\phi + \theta_1 + \theta_2 + \ldots + \theta_n \\ & + N_n \angle\xi_n + N_{n-1} H_n \angle\xi_{n-1} + \theta_n \\ & + N_{n-2} H_{n-1} H_n \angle\xi_{n-2} + \theta_{n-1} + \theta_n \ldots \\ & + N_1 H_2 H_3 \ldots H_n \angle\xi_1 + \theta_2 + \theta_3 + \theta_n \end{aligned} \qquad (6)$$

where the vector form $A\angle\alpha \equiv A(f)\angle\alpha(f)$.

Thus, the output $Y\angle\psi$ can be thought of as the sum of two vectors

$$Y\angle\psi = X\angle\phi\, H\angle\theta + N\angle\xi \qquad (7)$$

where $H\angle\theta = H_1 H_2 H_3 \ldots H_n \angle\theta_1 + \theta_2 + \ldots + \theta_n$

and $N\angle\xi = N_n \angle\xi_n + N_{n-1} H_n \angle\xi_{n-1} + \theta_n + \ldots$

$$+ N_1 H_2 H_3 H_4 \ldots H_n \angle\xi_1 + \theta_2 + \theta_3 + \ldots + \theta_n$$

Thus, the system of Figure 2 can be replaced by an equivalent network as shown in Figure 3(a), provided that we are only concerned with finding the transfer function between the input terminals where $X\angle\phi$ is applied and the output terminals where $Y\angle\psi$ is observed. The vector relationships are shown in Figure 3(b).

It can be shown that a similar input-output relationship exists for any of the "n" input ports. Hence, if $H\angle\theta$ can be measured, "n" similar measurements will describe the total set of network transfer functions. In some cases, only one transfer function is required, that which applies between the X-input and Y-output terminals. Now, the signals designated as $N_1\angle\xi_1$ through $N_n\angle\xi_n$ can be thought of as internal noise sources that may not be directly observable. In either case, the problem is essentially the same: Measure the transfer function in the presence of other spurious signals, which we will conveniently call noise.

The equivalent noise vector has introduced uncertainty in the amplitude and phase measurements of the output vector $Y\angle\psi$. The measurement procedure must minimize these effects.

An analogous problem arises when the impedance of a noisy network is measured. In this case, the input vector of Figure 3 and equation (7), $Y\angle\psi$, becomes a voltage or velocity vector, $N\angle\xi$ a voltage or velocity noise vector, and $X\angle\phi$ the current or force vector. The equation relating the various factors is:

$$Y\angle\psi = X\angle\phi\, Z\angle\theta + N\angle\xi \qquad (7a)$$

where $Z\angle\theta$ represents the impedance of the network to be measured.

With these differences in mind, the results and discussions that follow for estimating the transfer function will apply directly for estimation of the impedance function.

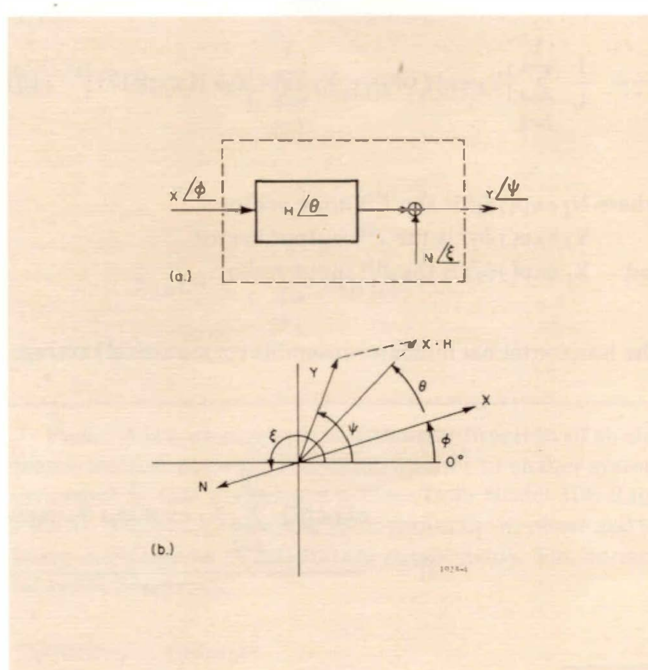


Figure 3. Noisy-network derivation.

E. A. Sloane received his BSEE degree from Northeastern University in 1951. He was a staff member at MIT Lincoln Laboratory, radar systems group, from 1951 to 1954; he joined Ampex Corp. as manager first for product planning, then for advanced technology planning, 1955-1965. Presently, he is Vice President and Technical Director of Time/Data Corporation, a GR subsidiary, which he helped found. Mr. Sloane is a member of Tau Beta Pi and Eta Kappa Nu.

Least-Mean-Square Estimation

Because the spurious signals are treated as a single equivalent noise vector, we can express it in terms of the transfer-function input and output as follows:

$$N\angle\xi = Y\angle\psi - X\angle\phi \; H\angle\theta \tag{8}$$

or $$N\exp(j\xi) = Y\exp(j\psi) - X\exp(j\phi)\,H\exp(j\theta)$$

The mean-squared value of this error after L* measurements will be:

$$\overline{\epsilon^2} = \frac{1}{L}\sum_{l=1}^{L}\left|N_l \exp(j\xi_l)\right|^2$$

$$= \frac{1}{L}\sum_{l=1}^{L}\left|Y_l \exp(j\psi_l) - X_l \exp(j\phi_l)\,H\exp(j\theta)\right|^2 \tag{9}$$

where $N_l \exp(j\xi_l)$ is the l^{th} noise vector

$Y_l \exp(j\psi_l)$ is the l^{th} output vector

and $X_l \exp(j\phi_l)$ is the l^{th} input vector.

The horizontal bar indicates ensemble (or statistical) average.

Performing the indicated operations in equation (9), we obtain

$$\overline{\epsilon^2} = \frac{1}{L}\sum_{l=1}^{L}\Big\{Y_l^2 + H^2 X_l^2 - H X_l Y_l \exp[j(\theta + \phi_l - \psi_l)] - H X_l Y_l \exp[-j(\theta + \phi_l - \psi_l)]\Big\} \tag{10}$$

Because we desire to minimize this error by choosing an appropriate estimator** for the gain, $\hat{H}$, and the phase angle, $\hat{\theta}$, we proceed by differentiating with respect to the phase angle and setting the derivative to zero. Thus,

$$\frac{\delta\overline{\epsilon^2}}{\delta\hat{\theta}} = -\frac{1}{L}\sum_{l=1}^{L} H\Big\{X_l Y_l \exp[j(\hat{\theta} + \phi_l - \psi_l)] - X_l Y_l \exp[-j(\hat{\theta} + \phi_l - \psi_l)]\Big\} = 0 \tag{11}$$

so that

$$\exp(2j\hat{\theta}) = \frac{\sum_{l=1}^{L} X_l Y_l \exp[-j(\phi_l - \psi_l)]}{\sum_{l=1}^{L} X_l Y_l \exp[+j(\phi_l - \psi_l)]} \tag{12a}$$

or $$\exp(2j\hat{\theta}) = \frac{\sum_{l=1}^{L} X_l \exp(-j\phi_l)\, Y_l \exp(+j\psi_l)}{\sum_{l=1}^{L} X_l \exp(+j\phi_l)\, Y_l \exp(-j\psi_l)}. \tag{12b}$$

The essential estimator for the phase angle of the transfer function involves the sum of appropriate forms of the product of the input vectors, $X_l \exp(-j\phi_l)$, and the output vectors, $Y_l \exp(+j\psi_l)$.

In a similar manner, by differentiating with respect to the gain function, H, we obtain the following least-mean-squared estimator:

$$\hat{H} = \frac{\exp(j\hat{\theta})\sum_{l=1}^{L} X_l \exp(j\phi_l)\, Y_l \exp(-j\psi_l) + \exp(-j\hat{\theta})\sum_{l=1}^{L} X_l \exp(-j\phi_l)\, Y_l \exp(+j\psi_l)}{2\sum_{l=1}^{L} X_l^2} \tag{13}$$

*Number of independent observations.

**An approximation to some value function.

Note that the estimate for the gain function, $\hat{H}$, depends on knowledge of the phase function, $\exp(j\hat{\theta})$, as well as of products of the input and output vectors.

Equation (13) can be restated as follows

$$\hat{H}\exp(j\hat{\theta}) = \frac{\exp(2j\hat{\theta})\sum_l X_l \exp(j\phi_l) Y_l \exp(-j\psi_l) + \sum_l X_l \exp(-j\phi_l) Y_l \exp(+j\psi_l)}{2\sum_l X_l^2} \quad (14)$$

so that substitution of equation (12b) yields

$$\hat{H}\exp(j\hat{\theta}) = \frac{\sum_l X_l \exp(-j\phi_l) Y_l \exp(+j\psi_l)}{\sum_l X_l^2} \quad (15)$$

Equation (15) represents the least-mean-squared estimate of the system transfer function.

Because Equation (15) may be expressed as

$$\hat{H}\exp(j\hat{\theta}) = \frac{\frac{1}{L}\sum_l X_l \exp(-j\phi_l) Y_l \exp(+j\psi_l)}{\frac{1}{L}\sum_l X_l^2} \quad (16)$$

the numerator can be recognized as the Cross-Spectral Density estimate $\hat{S}_{xy}$ of the input and output, or

$$\hat{S}_{xy} = \frac{1}{L}\sum_{l=1}^{L} X_l \exp(-j\phi_l) Y_l \exp(+j\psi_l), \quad (17)$$

while the denominator is the Auto-Spectral Density estimate $\hat{S}_{xx}$ of the input signal, or

$$\hat{S}_{xx} = \frac{1}{L}\sum_{l=1}^{L} X_l^2, \quad (18)$$

so that the transfer-function estimator becomes

$$\hat{H}\exp(j\hat{\theta}) = \frac{\hat{S}_{xy}(f)}{\hat{S}_{xx}(f)} \quad (19)$$

This is simply the ratio of the cross spectrum to the auto spectrum of the input function.

For the sake of completeness, we should mention that these vector functions may be expressed in terms of rectilinear coordinates representing in-phase and out-of-phase (co and quad) components by means of the following identities

$$\exp[\pm j\beta(f)] = \cos\beta(f) \pm j\sin\beta(f) \quad (20)$$

and

$$Z(f)\exp[\pm j\beta(f)] = Z(f)\cos\beta(f) \pm jZ(f)\sin\beta(f) = Z(\pm jf) \quad (21)$$

so that

$$H(jf) = \frac{\hat{S}_{xy}(f)}{\hat{S}_{xx}(f)} \quad (22)$$

and

$$\hat{S}_{xy}(f) = \frac{1}{L}\sum_{l=1}^{L} X_l(jf)\, Y_l(-jf) \quad (23)$$

$$\hat{S}_{xx}(f) = \frac{1}{L}\sum_{l=1}^{L} |X_l(jf)|^2 \quad (24)$$

Figure 4 is a photograph of a transfer function of an electromechanical network (vibration exciter or shaker system) estimated in this manner on a Time/Data Model 100 Rapid Fourier Analyzer. The upper trace shows the in-phase and the lower out-of-phase or quadrature components. The horizontal axis is frequency.

Reliability of Estimate

Finally, the error in estimation can readily be determined from equation (15) by substituting equation (8), so that

$$\hat{H}\exp(j\hat{\theta}) = \frac{\sum_l X_l \exp(-j\phi_l)\,[H\exp(+j\theta)\,X_l\exp(+j\phi_l) + N_l\exp(+j\psi_l)]}{\sum X_l^2} \quad (25)$$

or

$$\hat{H}\exp(j\hat{\theta}) = H\exp(j\theta) + \frac{\sum_l X_l N_l \exp[j(\psi_l - \phi_l)]}{\sum X_l^2} \quad (26)$$

Therefore, the error in estimation, η, introduced by this procedure, is

$$\eta = \frac{\sum_l X_l N_l \exp[j(\psi_l - \phi_l)]}{\sum_l X_l^2} \quad (27)$$

This tends to approach zero as the number of observations increases indefinitely, provided that the input and noise signals are uncorrelated, because the numerator would represent the sum of a large number of vectors having random phase angles ($\psi_l - \phi_l$), while the denominator would be a real positive definite quantity that increases linearly with the number of observations.

The mean-squared error, $\overline{\eta^2}$, in measurement can be shown to be

$$\overline{\eta^2}(f) = \frac{1}{L}\,\frac{S_{nn}(f)}{S_{xx}(f)} \quad (28)$$

where $S_{nn}(f)$ is the equivalent output-noise spectral density. The mean-squared error is inversely proportional to L, the number of independent observations entering into the estimate.

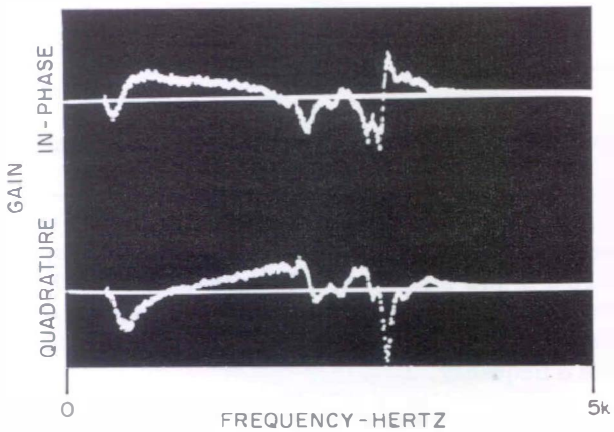


Figure 4. Transfer function of vibration-exciter system.

This error involves knowledge of the spectrum of noise; therefore, we must estimate the error by estimating the noise spectrum. The noise spectrum is approximately

$$\hat{S}_{nn}(f) = \frac{1}{L}\sum_{l=1}^{L} N_l^2(f) \quad (29)$$

and, by equations (9) and (16), becomes

$$\hat{S}_{nn}(f) = \hat{S}_{yy}(f) - \frac{|\hat{S}_{xy}(f)|^2}{\hat{S}_{xx}(f)} \quad (30)$$

$$\text{where } \hat{S}_{xy}(f) = \frac{1}{L}\sum X_l Y_l \exp[j(\psi_l - \phi_l)] \quad (31)$$

$$\hat{S}_{xx}(f) = \frac{1}{L}\sum X_l^2 \quad (32)$$

$$\hat{S}_{yy}(f) = \frac{1}{L}\sum Y_l^2 \quad (33)$$

The mean-squared error, in terms of measurable quantities, becomes

$$\overline{\eta^2} \approx \frac{1}{L}\,\frac{1}{\hat{S}_{xx}(f)}\left[\hat{S}_{yy}(f) - \frac{|\hat{S}_{xy}(f)|^2}{\hat{S}_{xx}(f)}\right], \quad (34)$$

$$\text{or } \overline{\eta^2} \approx \frac{1}{L}\,\frac{\hat{S}_{yy}(f)}{\hat{S}_{xx}(f)}\left[1 - \frac{|\hat{S}_{xy}(f)|^2}{S_{xx}(f)\,S_{yy}(f)}\right] \quad (35)$$

$$\text{or } \overline{\eta^2} \approx \frac{1}{L}\,\frac{\hat{S}_{yy}(f)}{\hat{S}_{xx}(f)}\,[1 - \Gamma(f)], \quad (36)$$

$$\text{where } \Gamma(f) = \frac{|\hat{S}_{xy}(f)|^2}{S_{xx}(f)\,S_{yy}(f)} = \frac{S_{yy}(f) - S_{nn}(f)}{S_{yy}(f)}. \quad (37)$$

From this it is apparent that

$$0 \lesssim \Gamma(f) \lesssim 1. \qquad (38)$$

The function, $\Gamma(f)$, is known as the Coherence Function. It is zero when the output spectrum is due entirely to noise ($S_{yy} = S_{nn}$), and unity when the system is noise-free ($S_{nn} = 0$). This is a popular measure for the reliability of a transfer-function estimate.

In summary, we have found that the estimator that minimized the effects of spurious signal or noise sources involved measurements of the magnitude and phase of the input and output signals for each frequency of interest. These measurements could, in principle, be obtained by means of voltmeters and a phase meter. However, if the number of frequency points that must be used to describe the transfer or impedance function is large, then more efficient estimates can be made by means of digital-signal processors using fast Fourier-transform techniques. The T/D Models 1923A Time-Series Analyzer, 1923B Real-Time Fast Fourier-Transform Analyzer, or 1923C Fast Fourier-Transform Analyzer, Figure 5, directly perform the operations indicated in equation (19), utilizing the Time/Data 90 Rapid Fourier Processor. The 1923A, for example, will perform a transfer-function analysis for 128 equally spaced frequencies in less than 10 milliseconds per complete estimate.

– E. A. Sloane

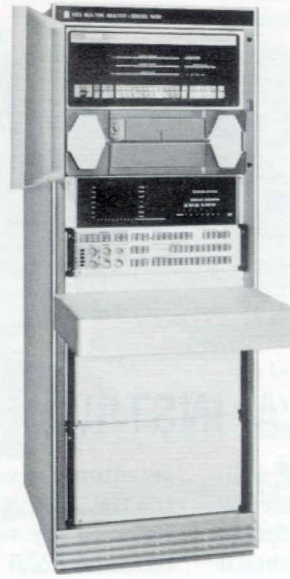


Figure 5. T/D 1923-C Real-Time Analyzer.

For Further Information

Bendat, J. S., and Piersol, A. G., *Measurement and Analysis of Random Data*, John Wiley and Sons, 1966.

Lee, Y. W., *Statistical Communications Theory*, John Wiley and Sons, 1966.

Sloane, E. A., "An Introduction to Time-Series Analysis," *Monographs I, II, and III*, Time/Data Corporation.

The Honorable Society

C. E. White

A. P. G. Peterson

On June 16, 1970, the National Conference of Standards Laboratories presented its first Awards for Outstanding Service to three members of the organization. One of the recipients was Charles E. White of General Radio. While presenting the award plaque, NCSL Chairman J. L. Hayes made the following citation:

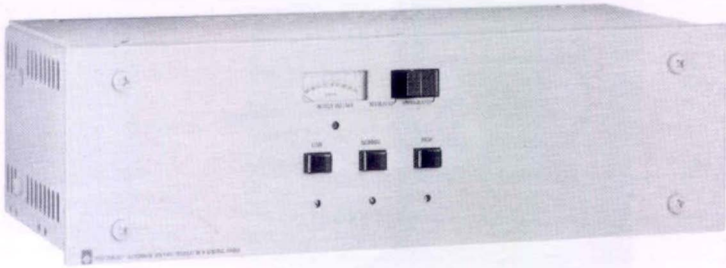
"Mr. Charles E. White has unselfishly devoted his time and energies to formulating, editing, and sustaining the operation of the *NCSL Newsletter* for the past eight and one-half years. Much of the credit for the growth of this publicity and information media is reflected upon Mr. White, who served as chairman of the NCSL Newsletter Committee and Editor of the *Newsletter*."

The Audio Engineering Society has announced the impending award of Fellow membership to Dr. A. P. G. Peterson of General Radio. Presentation will take place at the annual Awards Banquet in New York on 14 October 1970.

Dr. Peterson is well known in the field of acoustics. He received the John H. Potts Memorial Award from the AES, in 1968, for outstanding achievement in the field of audio engineering. He is, also, a Fellow of the Institute of Electrical and Electronic Engineers and of the Acoustical Society of America.

Deviations from Accuracy

We are aware of two small errors that crept into the March-June, 1970 issue. The first, on page 5, should have translated the test pressure of 1000 microns to 1 mmHg. The second, on page 18, should have specified that switching time of the GR frequency synthesizer had been reduced to 200 μs (not 200 ms). Sorry!



NOT A CINDERELLA INSTRUMENT!

The fate of ordinary, practical, well-known, and widely-used instrumentation apparently is relegation to drudgery work and to near-obscurity when it comes to publicizing new models. The apparent lack of new-design innovation is compounded by the manner in which the general public disregards ordinary but essential instruments. We've decided to challenge this attitude by writing about the new GR 1592 Variac® automatic voltage regulators with the attitude of "How can you do without it?"

If you are a typical reader, you have control over, or access to, one or more racks of test equipment. Conceivably, each instrument is well engineered and has its own regulated power supply. Consequently, you foresee no need for a second, or master, regulator. Consider for one moment, however, that all your instruments may operate satisfactorily with an ac supply voltage of 105 volts. Modern voltage regulators usually control any voltage above the nominal 105 volts by means of a semi-conductor power-dissipating circuit to drop the voltage close to the nominal value.

In the case cited above, under normal circumstances a fair amount of heat is dissipated within the rack of instruments. Expand the number of racks and you develop the need for an air-conditioned environment to keep the mean-time-between-failure rate of the instruments from increasing rapidly. It would appear to be good logic, therefore, to reduce the supply voltage to 105 volts nominal, and to maintain continuous operation at that level. Such a step would reduce the amount of heat generated and greatly increase the useful life of the instruments. Installation of an automatic voltage regulator such as the GR 1592 maintains the instrumentation supply voltage at the desired *low* operating point.

Note that *all* voltage regulators are *not* capable of doing this. Constant-voltage transformers and reactor-type regulators operate solely to maintain the instrumentation supply voltage at the normal voltage established by the power company's distribution transformer. The GR 1592 is an electro-mechanical regulator whose output is controlled by a servo-driven Variac adjustable autotransformer of proven capability.

There is a more important point to be considered: the GR 1592 does not introduce distortion into the instrument supply voltage – a factor ignored too often by customers of regulators. In a previous *Experimenter* article,[1] we mentioned that the GR electro-mechanical regulator could track the average and peak values of supply voltage while actually detecting the rms value. This feature has significance in a number of situations. Take, for example, a capacitive-input dc power supply with a light load. Such a unit responds to peak supply voltages. A 3% distorted output from the regulator could cause as much as 3% change in the dc power-supply output, even though the regulator held to a specified 0.1% limit of deviation from a nominally rated supply voltage.

If the regulator were in control of a thermal device, which responds to the rms value of the supply voltage, distorted regulator output would affect operation of the thermal unit in a manner similar to the preceding example. When the regulator is used to control heavily-loaded capacitive-input instruments, or inductive-input power supplies, or just plain ordinary mechanical systems (all of which respond to the average values of the supply voltage), distorted output from the regulator can quickly lead to instrument operation inferior to rated performance.

The advent of digital instrumentation has created an awareness, among instrument users, of the devastating effects of spikes or sharp peaks in the supply voltage. False triggering of digital circuits is commonplace when supply voltages are used in common with distortion-producing instrumentation. Use of the GR 1592 as a buffering voltage supply unit to a block of digital instruments helps to reduce false digital outputs.

The GR 1592 can even be considered as a tool to help mitigate "brown-outs", so widely predicted by metropolitan power companies during peak-power-load seasons. If your local power company is forced to lower supply voltages drastically, use of the regulator will assure continued and satisfactory operation of instrumentation.

For readers with problems of supply for illumination devices, plating baths, or similar applications drastically affected by line-voltage variations, the GR 1592 is available for loads up to 10 kVA. Lighter load demands undoubtedly could be met by the 2-kVA model.

Complete specifications for the models listed below are available in GR Catalog U. A pamphlet describing current GR models of voltage regulators is available to readers. Address your request to:

Editor, *GR Experimenter*
300 Baker Avenue
Concord, Massachusetts 01742

Design responsibility for the GR 1592 was shared by C. G. Chitouras and W. A. Montague.

Catalog Number	Description	Price
	1592 Variac® automatic voltage regulator	
1592-9700	120-V ±10% input	**$525.00**
1592-9701	120-V ±20% input	**525.00**
1592-9702	230/240-V ±5% input	**525.00**
1592-9703	230/240 V ±10% input	**525.00**
1592-9704	230/240-V ±20% input	**525.00**

Prices net FOB Concord, MA, USA.
Subject to quantity discount.

[1] Chitouras, C., "Considerations In The Choice of a Line-Voltage Regulator, "*GR Experimenter*, October 1967.

GR 1541 Multiflash Generator

NEW SHOES FOR AN OLD WORKHORSE

For a period of about 38* years, the principle of observing high-speed motion by means of electronic stroboscopes[1] has been implemented by portable GR equipment of various types. There has been wide acceptance by industry[2,3,4] of this instrumentation, capable of "stopping" fast motion without physical contact for quick analysis and of preserving the stopped-motion on film for later study.[5]

A technique, familiar to photographers, which has not received wide attention in engineering and research activities, is most useful in stopping the motion of extremely high-velocity actions without the use of high-speed cameras. This is the multi-flash or burst-flash technique by which a single sheet of film is exposed, in consecutive order, by a series of strobe flashes.

The same technique can be expanded, as test requirements become more complex, to provide for a pulse burst from a single stroboscope (Figure 1a) or a burst of individual flashes from multiple strobes (Figure 1b). It provides equivalent-shutter speeds of a microsecond to conventional ciné and high-speed motion picture cameras when the new GR 1541 Multiflash Generator is used.

The implementing system uses a stroboscope, multiflash generator, and still camera to provide frozen action on a single Polaroid** film within 15 seconds of the event. A series of complete images, each uniquely positioned on the film in time and space sequence, is available for study and action.

*See page 18 for a partial reprint of the original GR strobe article.

**Registered trademark of the Polaroid Corp.

A Glance at the Features

Among the numerous features of the GR 1541 generator are:

- Flash bursts, adjustable in numbers and intervals
- Versatile trigger circuit, designed to accept a variety of inputs in terms of signals and connectors
- Flash intervals that can be calibrated
- Adaptability to existing stroboscopes
- Small, light, and rugged construction
- Highest intensity retained at 10-μs intervals
- Burst mode provides for initial signal to activate contact-bounce and noise-rejection circuits.

"Stopped" In Its Tracks

Many applications in motion-analysis work have been developed during the years that the stroboscope has been

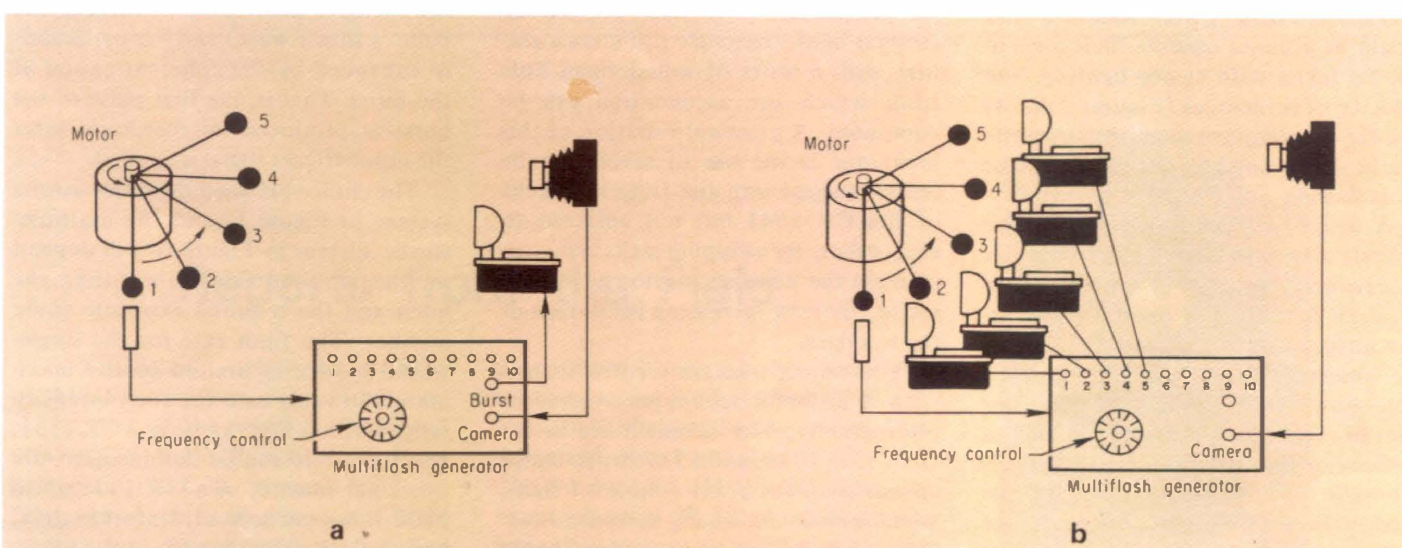
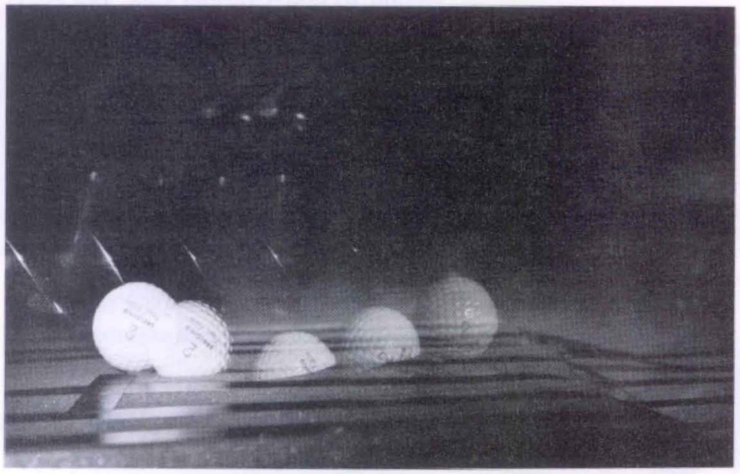


Figure 1. Flash-burst techniques. a. Single strobe unit; b. Multistrobe units.



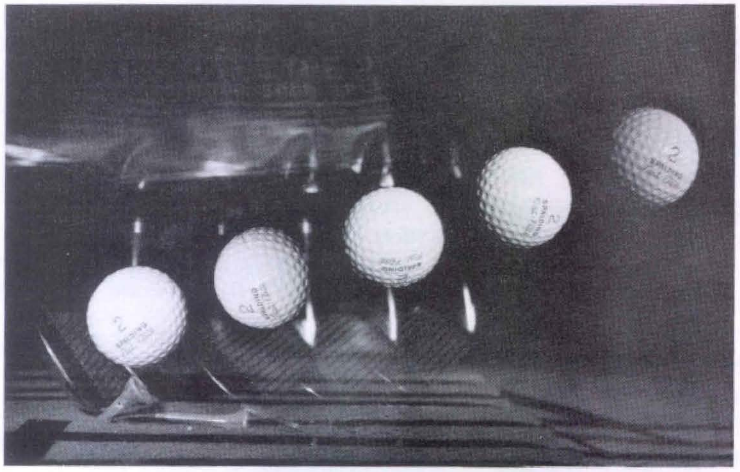


Figure 2. Golfball stroking analysis – upper, topping; lower, good lift.

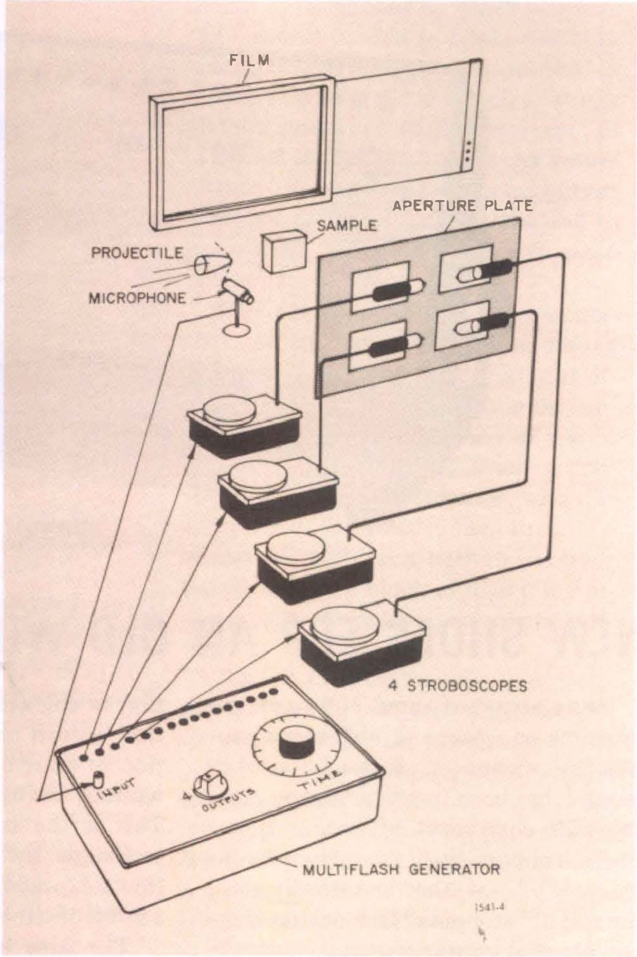


Figure 3. Instrument arrangement for projectile-motion study.

with us. Some common uses include studies of sports equipment in action, such as the impact and flight of golf balls (Figure 2) or the motion of toppling bowling pins. Athletes and would-be athletes observe their form in photos taken with strobe lighting. The velocity of projectiles (Figures 3 and 4) and the acceleration of machinery units can be determined by the use of two or more flashes.

A desirable feature in pulsed strobe lighting is that the moving object under observation get out of its own way between flashes. If it does not, successive images will overlap and "wash out" the observed action. Overlap is permissible, however, if pictures are taken for data or record purposes only, inasmuch as three or four overlapping images can normally be resolved. If color film is used, with different colored filters over each strobe light (Figure lb), overlapped-image recognition is substantially increased.

You can observe the acceleration of a highly reflective, rotating shaft by means of another technique. The end of the shaft is painted flat black with a single peripheral white dot. Viewed by strobed light, the single dot gives a picture with a series of well-defined dots from which the acceleration can be computed. A practical variation of this technique is the use of several strobe units, plugged into the triggering jacks of the GR 1541 but not adjacent to each other. By skipping jacks, you can increase the point separation at low velocity, thereby increasing resolution of the test data.

You can separate consecutive images with a different technique – shadow photography. The compact arc in the GR 1531, 1538, and 1539 strobe lamps approximates a point source of light, which can cast unusually sharp shadows. By use of multiple, separated strobes in the system of Figure 3, the images at each succeeding interval of time fall on different areas of the film to produce a record, like that of the bullet striking the steel spring in Figure 4. Synchronization of this system is quite simple (an inexpensive microphone detects the bullet's shock wave) and it is particularly enhanced by the coherent nature of the burst. That is, the first pulse of the burst is produced microseconds after the input-trigger signal is applied.

The choice between the single-strobe system in Figure la and the multiple-strobe system in Figure lb will depend on the required flash rate during the burst and the required exposure guide number. The flash rate for the single-strobe system is limited to the maximum allowable rate for each intensity range setting. For example, a GR 1531, 1539, or 1540 can be flashed up to 400 times per second, or a GR 1538 up to 2500 times per second. Unfortunately, higher flash rates require lower intensity settings; consequently, larger lens apertures are required, resulting in re-

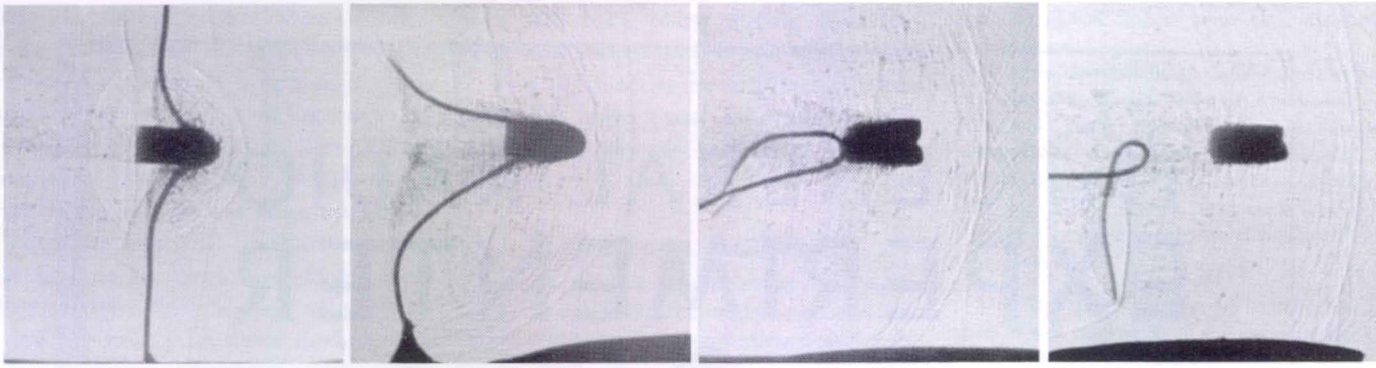


Figure 4. Multiple-flash shadowgraph sequence.

duced depth of field. This decrease in exposure corresponds to approximately 2-1/2 f-stops per intensity-range step for GR stroboscopes. When multiple strobes are used, each may be set to its highest intensity settings, thus recovering as much as 5 to 7-1/2 f-stops, with consequently increased depth of field. Each strobe is single-flashed, so that the resulting maximum burst frequency is 100,000 per second, limited only by the GR 1541 generator.

Using strobe light as the shutter eliminates blur and several distortions present even in high-speed cameras and often provides ample lighting at considerably reduced cost, weight, and line input power. For example, four GR 1538 stroboscopes and a 1541 generator can be used to produce 10,000 flashes-per-second light, about the upper speed limit for a good full-frame high-speed camera.

Some Technical Points

The GR 1541 Multiflash Generator can provide for burst groups of two to sixteen flashes, with separation between flashes continuously adjustable from 10 microseconds to 100 milliseconds. Trigger circuits in the instrument provide for a variety of input-signal sources, electrical, photoelectrical, and electromechanical. The trigger circuit is designed to reject noise or signals that occur after the initiating signal. Flash-interval control has a basic error limitation of 3%, but provision is made for a calibration signal to an external electronic counter if greater accuracy is required.

REFERENCES

1. Lamson, H. W., "The Stroboscope," *General Radio Experimenter,* December 1932.

2. Fitzmorris, M. J., et al, "New Eyes for Modern Industry," *General Radio Experimenter,* September 1960.

3. Fitzmorris, M. J., "Flash-Delay Unit Simplifies Motion Analysis in High-Speed Machines," *GR Experimenter,* August 1963.

4. Holtje, M. C., "Flash! A New Strobotac® electronic stroboscope," *GR Experimenter,* April 1966.

5. Miller, C. E., "Detailed Viewing in Ambient Brightness," *GR Experimenter,* September/October 1969.

Development of the GR 1541 was by C. E. Miller and W. F. Rogers, who also collaborated on the above material.

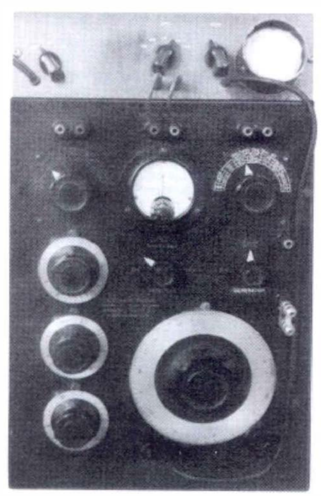
Specifications for the GR 1541 Multiflash Generator are included as a tear sheet in the back of this issue.

Catalog Number	Description	Price
1541-9701	**1541 Multiflash Generator**	**$675.00**
1541-9601	**Cable Assembly**	**7.50**

Prices net FOB Concord, MA, USA. Subject to quantity discount.

Reports from the Field

As a nostalgic touch in the last issue of the *Experimenter*, we included a reproduction of the original article that announced the GR 650 Impedance Bridge in 1933. It brought reactions from the readers, all pleasant. One in particular, Everett Mehner of San Diego, California, was pleased to describe the manner in which he had modernized his own bridge, purchased on the surplus market three years ago. He used the battery box to house a transistorized signal generator (100, 400, 1k, 4k, and 10k Hertz) and an oscilloscope display. The latter feature permits the operator to balance null points quite simply and visually.



The GENERAL RADIO EXPERIMENTER

VOL. VII. No. 7 — DECEMBER, 1932

ELECTRICAL COMMUNICATIONS TECHNIQUE AND ITS APPLICATIONS IN ALLIED FIELDS

THE STROBOSCOPE

> THE quickness of the hand deceives the eye. But the eye knows a trick or two, and, aided by ingenious mechanisms, it is not deceived by the gyrations of machinery at far higher speeds than the trickster's hand achieves. Hence the stroboscope, which is not new, and the Edgerton* stroboscope, which is.
>
> Stroboscopes and their applications are described herewith. The Edgerton stroboscope on page 5.

THE stroboscope consists fundamentally of a device which permits intermittent observations, either visual or photographic, of a moving object in such a manner as to reduce the speed of, or stop, the motion.

The slow-motion picture is a familiar example of the interesting and profitable information which may be derived from a leisurely study of events which necessarily take place at a high rate of speed. The tennis player cannot slow the championship stroke to accommodate the laggard eye of the novice, but the camera can, and the motion picture camera is a stroboscope, but not all stroboscopes are cameras.

The camera shutter, operating at high speed, chops up the action into a number of small elements, so short that movement is not apparent in any one. The film can then be projected at normal speed with results that are instructive, or even backward with results that may be amusing. The function of the shutter is to exclude light from the film except for brief flashes. It seems reasonable that the same result can be obtained by shutting off the light from the object, except for brief flashes. This is the nature of the second style of stroboscope, of which the Edgerton type is the outstanding example. Obviously this type of stroboscope is well adapted for visual observations. Photography must still be used, if a non-repeated event is viewed, to store the elementary views and to release them later at a rate that the eye and mind can follow.

Consider, however, an indefatigable tennis player who repeats his stroke.

*The Edgerton Stroboscope is a development of Prof. Harold E. Edgerton, Massachusetts Institute of Technology.

identically, one thousand times a minute in a darkened room. If the light be flashed on him at a constant rate, exactly equal to his stroking rate, he will appear as though motionless under continuous illumination. If the flash speed be slightly slower than his stroking rate, his arm will be illuminated a little farther along in the stroke each time the light flashes and, as the eye retains the image between flashes, the madly stroking player will seem leisurely, and a single stroke can be spread over a minute if desired.

Humans, tennis playing or otherwise, cannot repeat uniform cycles at any such speed. Machines can, and wherever complicated machines are designed, built, or used, the ability to watch their operation in slow motion without photography is a boon.

The stroboscope permits stopping the motion of the machine (visually) for examination of machine or product at any part of its operating cycle while the grommets flow into the hoppers at undiminished speed. Or, perhaps, a squeaking clutch, a vibrating shaft, or a chattering valve spring stands between a new model and a waiting public—which will not wait long. A slow motion study will show the trouble, or the primary motion may be stopped and the vibrating member made as conspicuous as a mosquito-brushing hand at formal guard mount.

Sometimes the transient movement or vibration takes place at too high a speed for the eye even with the primary motion stopped. Here photography is resorted to for a second slowing down of the transient.

A little consideration of what is being done by the stroboscope is sufficient to set up the requirements of a satisfactory one.

An accurate means of timing the flash and a prompt and accurate response to the flash control are essential, otherwise the object will be viewed at irregular intervals, and vibrations not present in the object viewed will be introduced.

The flash must be of extremely short duration. Otherwise appreciable motion will take place during illumination, and blurring of detail will result.

The light must be brilliant. Otherwise the room must be made entirely dark, and details will not be seen clearly.

Stroboscope Arithmetic

Suppose that the object to be observed is executing uniformly R complete cycles of motion in unit time. Suppose further that the object is either viewed through a shutter opening for F brief, uniformly timed intervals, or is illuminated by F uniform instantaneous flashes of light in unit time. Then, if

$$R = nF \qquad (1)$$

where n is an integral number, it will be evident that each point of the object will be in exactly the same position in its cycle of motion at each observation, resulting in what we shall designate as a condition of "perfect" synchronism. Accordingly, all apparent motion of the body will be arrested, so that it will appear to be stationary at some particular phase in its cycle of motion, provided that the opening of the shutter or the flash of the lamp is of extremely short duration. If this interval of observation is of sufficient duration, the moving object, even when viewed stroboscopically, will appear blurred in outline, since each point of the body executes a perceptible amount of motion during the interval of observation.

It is further evident that the phase of the observed position of the object in its cycle of motion may be controlled at will merely by shifting the phase of the synchronous shutter or light flash with respect to the motion.

The special case of perfect synchronism, in which the frequency of motion and of observation are identical, is known as "fundamental" synchronism.

If n is greater than 1, the object will be observed only at every nth cycle of motion, so that the integrated illumination is reduced to the fractional amount $1/n$ times the illumination at fundamental synchronism.

Although any condition of perfect synchronism will completely arrest the motion, it is obviously desirable to work at the condition of fundamental synchronism.

If, on the other hand,

$$F = kR \qquad (2)$$

where k is any integral number greater than 1, then each point of the object will be visible k times per cycle of motion and will, accordingly, be observed successively at k points equally spaced, in time, throughout the cycle of motion. Such a condition, which is known as "partial" synchronism, while apparently arresting the motion of the object, is not, in general, satisfactory for visual stroboscopic observations. For example, a rotating disc having one radial line is seen as a disc with k radial lines.

A more distinct image is obtained at partial synchronism if the body is composed of mk identical parts equally spaced, in time, throughout the cycle of motion, *e.g.*, by a wheel having $P = mk$ spokes. Further, it can readily be shown that such a wheel will appear as a stationary wheel having P spokes whenever

$$PR = nF \qquad (3)$$

On the other hand, the wheel having P spokes will appear as a stationary wheel having nP spokes whenever

$$nPR = F \qquad (4)$$

Reference to equation (3) shows that there are, theoretically, an infinite number of values of R or of F for which a wheel of P spokes will be seen as a stationary wheel of P spokes. The larger the value of P, the greater will be the number of these partial synchronisms which occur within a given range of values of R or F. These facts are of importance in using the stroboscope to determine the frequency or speed of cyclic motions.

We have so far analyzed the fundamental laws of the stroboscope for conditions of exact synchronism, either partial or perfect. Consider now the case where the cyclic frequency of motion is slightly greater than an integral multiple of the frequency of observation—

$$R = nF + S \qquad (5)$$

where S is small compared to R. This means that the moving object will execute slightly more than n cycles of motion during the interval between observations so that the phase at which it is seen stroboscopically will continually advance. The object will therefore appear to move at a slow cyclic frequency of

$$S = R - nF \qquad (5a)$$

cycles in unit time and to travel in the same direction as the object is actually moving.

Conversely, if the cyclic frequency is slightly less than an integral multiple of the frequency of observation the phase at which the object is seen stroboscopically will continually recede so that the object will appear to move at a slow cyclic frequency in a direction opposite to the true motion:

$$S = nF - R \qquad (6)$$

The slow stroboscopic motion which can be obtained in this manner, and which can be adjusted to become a very small fraction of the true speed, makes the stroboscope extremely valuable in watching the cycle of motion of machinery running at speeds too high to be followed with the unaided eye.

The frequency of stroboscopic motion, S, may be made as slow as desired. On the other hand if S is increased above a certain limit the observed motion becomes intermittent and less satisfactory for purposes of visual study.

— Horatio W. Lamson

another new product

PROGRAMMABLE DECADE RESISTOR

The GR 1435 Programmable Decade Resistor was designed for maximum customer-use flexibility consistent with accuracy and cost. The basic instrument covers the five-decade span from 10-Ω to 100-kΩ per step, with each decade a plug-in board. Mechanical and electrical provision has been made to allow simple conversion to a six- or seven-decade instrument, should the need arise. Reed switches used throughout the instrument are of the miniature mercury-wetted type, for low and repeatable zero resistance as well as bounce-free operation. The high and low terminals of the resistors are isolated from ground; this permits use where a floating resistor is required.

Three distinct modes of operation are provided for the user's convenience:

Manual Mode The desired resistance is set on the front-panel dials, just as one would set a conventional decade resistor. This is useful, for example, when you are making accuracy checks on the GR 1435, to determine how many decades need be remotely controlled for a particular application, or when you are manually checking proper system operation during initial set-up stages.

Manual/Remote Mode Some of the decades may be set to the "R" position on the front-panel dials and be remotely controlled, while the remaining dials are set to a particular value of resistance and held constant. This has the advantage of requiring four less control lines for every decade which is manually set.

Remote Mode You can select this mode by turning all decade dials to the "R" position; by turning the power switch to the "REMOTE" position; or by applying logic "0" to pin V of the rear-panel connector. Resistance is set by application of negative true 1-2-4-8 BCD signals at standard DTL or TTL levels, or contact closures to ground, to each decade via the 36-pin rear-panel connector.

A feature that deserves special mention is the ability to short or open the decade resistor terminals remotely. Grounding pin 18 of the 36-pin rear-panel connector shorts the resistor terminals, while grounding pin 17 opens the resistor terminals. This is particularly useful if discontinuities are objectionable, which may exist when resistance settings are changed (the worst case being the change between "7" and "8" where all four reeds change state). With proper timing, the resistor terminals could be either opened or shorted during this switching interval, whichever is more pertinent to the application.

A few typical applications are illustrated in the figures, ranging from a simple programmable amplifier load to a programmable oscillator or time constant.

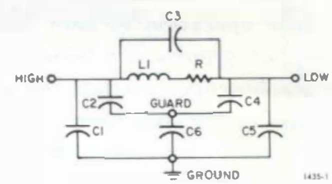
Catalog Number	Description	Price
	1435 Programmable Decade Resistor	
1435-9700	Bench Model	**$750.00**
1435-9701	Rack Model	**730.00**

Prices net FOB Concord, MA, USA. Subject to quantity discount.

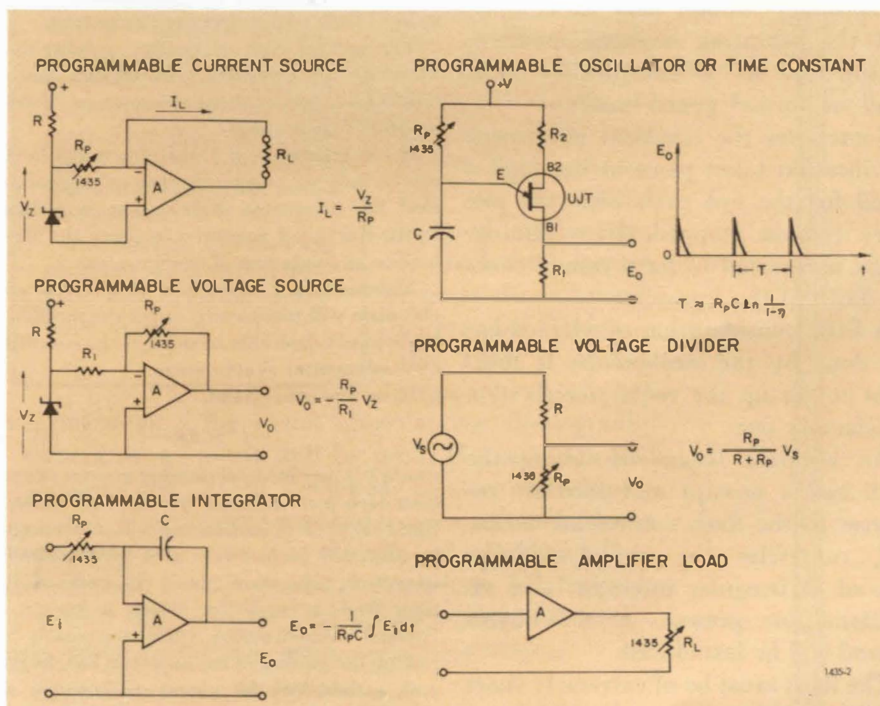
This instrument was designed by Peter Gray of GR's Component and Network Testing Group, who contributed the material for this article.

Complete specifications for the GR 1435 are in GR Catalog U; minor revisions are shown below.

Frequency Characteristics: At high-resistance values, frequency characteristics depend mainly on capacitances and on the type of connections used (2- or 3-terminal, grounded or guarded). At low resistance values, they depend mainly on the inductance. Calculations based on values shown should give approximate series-resistance error.



Parameter	Decade Resistance R = 100 kΩ	R = 1 MΩ
C1	19 pF	11 pF
C2	76 pF	23 pF
C3	19 pF	16 pF
C4	247 pF	276 pF
C5	46 pF	51 pF
C6	1606 pF	1606 pF
L1	23 μH	23 μH



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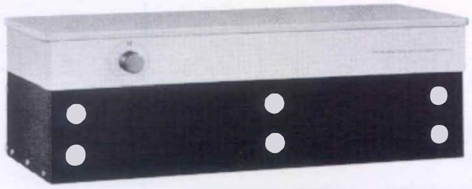
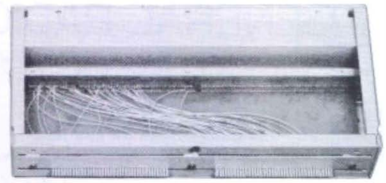


Universal Device Adaptor.



Underside of 1790-9603 Standard Device Adaptor Kit showing socket holes and typical wire-wrapped connections.

GR 1790—MORE VERSATILITY AND CAPACITY

Versatility is the name for the universal device adaptor recently made available to GR 1790 customers.

The logic-circuit analyzer has proven itself facile enough to handle most logic-circuit test problems presented it simply by means of suitably adapted interfacing.

GR engineers incorporated several important principles in the design of the interface adaptors. Connections within the adaptor are made simply, by wire wrapping to terminals mounted on printed boards. Auxiliary control or monitoring circuitry and loads are easily connected within the adaptors.

Customers are given the choice of a standard adaptor or of a universal adaptor. In the universal adaptors, input and output connections can be determined by the test program; this permits acceptance of a greater variety of devices as well as providing checks of outputs and inputs. Use of the universal adaptor permits tests for shorted inputs.

The adaptors are easily inserted into or removed from the analyzer by action of a single lever. Provision is made on one standard board to mount sockets in rows 0.250 inch apart and spaced at 0.125-inch internals. Adaptors are available with 24, 48, 72, and 96 inputs and 48, 72, 96, 120, and 144 outputs in predetermined combinations.

Complete specification details for the GR 1790 adaptors are available on the tear sheet at the back of this issue.

As customers for the standard GR 1790 Logic-Circuit Analyzer become familiar with its operation, we anticipate their needs will grow to expand its application to more complex tests. Or, a need for expanded memory storage will be evident as test programs lengthen. It is even possible that originally limited funding for capital expenditures may be increased as the savings, made possible by the GR 1790 in action, are brought to management's attention. For any of these reasons, GR is prepared to help its customers expand their standard analyzers by supplying and installing several retrofit kits.

Kits are available in two basic formats – one to expand memory by 50 times (Option 2) and the other to provide capability for the addition of programmable logic levels and programmable power supplies (Option 3). Both options are available in 50-Hz as well as 60-Hz versions, to accommodate overseas customers. Options are also available separately or combined. Neither option requires more physical space.

The kits will be installed by GR district office service-department personnel. Training required for operation with Option 2 will be provided the customer; no further training is required for Option 3.

Complete specification details for the GR 1790 options are available on the tear sheet at the back of this issue.

Description	Price
1790 Logic-Circuit Analyzer	
Retrofit Kits, for installation of options in the field by GR personnel	
Kit 2AK for Additional Memory, 60-Hz line	$10,500.00
Kit 2BK for Additional Memory, 50-Hz line	10,600.00
Kit 3K for Programmable Levels, 50 to 60 Hz line	13,500.00
Kit 2A-3K for both options, 60-Hz line	23,000.00
Kit 2B-3K for both options, 50-Hz line	23,100.00

	Dedicated Inputs	Dedicated Outputs	Programmable Inputs/Outputs	Price
Standard Device Adaptor Kits				
1790-9601 no socket holes	72	72	—	$ 130.00
1790-9602 no socket holes	96	144	—	195.00
1790-9603 socket holes	72	72	—	135.00
1790-9604 socket holes	96	144	—	195.00
Universal Device Adaptors				
1790-9605 no socket holes	72	120	24	600.00
1790-9606 no socket holes	48	96	48	825.00
1790-9607 no socket holes	24	72	72	1000.00
1790-9608 no socket holes	—	48	96	1350.00

Prices net FOB Concord, MA, USA.
Subject to quantity discount.

Recent Technical Articles by GR Personnel

"The Human Factor in Precise Measurements," C. E. White, *Measurements and Data*, March-April 1970.*

"Semi-Automatic DC-DVM Calibrator," R. P. Anderson, *Measurements & Data*, May-June 1970.**

"Computer Aids Redundant Logic Search," G. R. Partridge, *Electronic Design News (EDN)*, 15 June 1970.**

"Planning Investments in Research and Development," W. D. Hill, *Managerial Planning*, July/August 1970.*

"A Noise Exposure Meter," G. R. Partridge, to be presented 3-6 November, Acoustical Society of America.**

"Approximate Transfer Characteristics of a Condenser Microphone with Diaphragm Stretched Over Raised Points of the Backplate," S. V. Djuric, to be presented 3-6 November, Acoustical Society of America.**

*Reprints available from Editor – *Experimenter*, General Radio.
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